When does variety increase with quality?

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Abstract

Casual empiricism suggests higher quality is associated with greater variety. However, recent theoretical and empirical research has either not considered this link, or has been unable to establish unambiguous predictions about the relationship between quality and variety. In this paper we develop a simple model, which predicts that for low qualities variety should be positively correlated with quality and we establish conditions under which variety will either increase or decrease with quality at higher quality levels. The producer uses variety to increase the profitability of price discrimination across different qualities, by increasing the likelihood consumers choose high price products among products yielding the same utility. We show that the number of varieties offered by the monopolist is greater than the social optimum. The predictions of the model are supported by an empirical analysis of the market for cars. A wide range of car manufacturers are found to offer a hump-shaped distribution of varieties.

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1 Introduction

A casual look at the shelves of a supermarket or at producer web sites reveals that many goods come in multiple “flavors” as part of a product line. Moreover, higher quality products often have a larger number of flavors. Branded products almost always have more varieties than generics. For example, there are many more varieties of the premium Pickwick tea than of a basic Lipton brand. Furthermore, we demonstrate in this paper that this also holds for most of the product lines offered by car manufacturers. This phenomenon is not explained by the extensive theoretical literature examining the interaction between quality and variety in imperfectly competitive markets where consumers' tastes are differentiated along one vertical dimension, quality, and one horizontal dimension, flavor.[1] In such models the correlation between quality and variety depends on the way they enter the consumer's utility function, their joint distribution, and the market structure. Therefore, it is impossible to come up with unambiguous predictions suitable for estimation or testing.

Recent work by Draganska and Jain (2005) models varieties, in product-lines, as horizontal differentiation but does not explicitly consider vertical differentiation highlighted in models of price discrimination. Hence, the link between quality and variety is not analyzed.

We adapt a standard probabilistic choice model to demonstrate there tends to be a positive relationship between quality and variety. In the model, a second degree price-discriminating monopolist offers a menu of products of increasing quality. Consumers, modelled as caring about quality, but indifferent between flavors of the same quality, respond by randomizing across products offering the same utility. The monopolist increases the profitability of price discrimination by offering more flavors at higher qualities. This increases both the likelihood consumers choose high-price products when randomizing and the expected profits. However, if the market is thin at high quality levels, the profit maximizing number of varieties falls again. In this case, the distribution of varieties is hump shaped.

These predictions are tested using data on the product-lines of seventeen large interna-

[1]See, for example, Ansari, Economides, and Steckel (1998), Neven and Thisse (1990), and Shugan (1989).

tional car companies in the Australian market. We perform a dip test to determine most makes have a unimodal distribution of varieties. Most of the remaining makes are shown to have one large mode. We then estimate the flexible beta density functions for each make and conclude the densities for most makes are both unimodal and hump-shaped. It is notable this holds for makes from the discount to the premium end of the market. To determine if this pattern of varieties is an example of flavor proliferation, we test if the distribution of varieties offered is statistically significantly different from that consistent with the distribution of demand. A set of Kolmogorov-Smirnov tests suggest the distribution of varieties and demand differ for all but the cheapest makes in a way consistent with flavor proliferation.

One might object that most consumer product markets, including cars, are oligopolistic rather than monopolistic. However, the model can be generalized to this case along the lines of Champsaur and Rochet (1989). It can be shown that under some reasonable conditions there exists an equilibrium, where each producer specializes on a particular range of qualities. Qualitatively, the outcome is similar to the monopoly outcome. Proliferation of varieties in the oligopolistic case will be due to two effects: competition between the producers and price discrimination between consumers of different types who buy from the same producer. We find it interesting from the theoretical perspective that the second effect alone can lead to strong excessive flavor proliferation even if a consumer's choice is almost completely determined by the vertical characteristics of the product.

This paper makes two main contributions to the literature. First, the model provides a new explanation for observing different types of correlations between quality and variety. These correlations result from firms attempting to price discriminate rather than, as assumed earlier, solely from the distribution of preferences or costs — hypotheses that are notoriously difficult to test. Second, the generality of the hump-shaped distribution of varieties offered within the car market suggests similar distributions of product varieties may occur in other markets. Finally, also we show that the monopolist can produce a greater than optimal number of varieties if there is vertical heterogeneity and there is a relatively small amount of randomness in consumer behavior.[2] Most existing literature tends to find that the number of varieties produced by a monopolist tends to be less than optimal (Lancaster, 1990).[3]

The paper is organized in the following way. In Section 2 we provide some preliminary evidence to demonstrate a link between quality and variety for the car market. In Section 3 we introduce the simplest possible probabilistic choice model of consumer behavior with which to analyze the effect of increasing product variety on the profitability of price discrimination – the Luce model (Luce, 1959). The section closes by introducing the concept of a nearly deterministic consumer. In Section 4 we demonstrate that in a world populated by nearly deterministic consumers, the profitability of price discrimination will be lower compared with that predicted by the standard two-type screening model with fully deterministic consumers. In Section 5 we propose a way to overcome this problem by introducing multiple flavors for each quality level. In Section 6 we extend the model of Section 5 to multiple types and generate predictions about the relationship between quality and variety. Section 7 empirically analyzes these predictions using data from the Australian car market. Section 8 concludes.

2 Model proliferation in the car market

In this section, we provide some preliminary evidence on flavor proliferation from the car market. A comparison of luxury, medium and small models of cars demonstrates there are relatively more types of luxury cars on offer despite luxury cars sales being less than 10% of the market. We then argue that this pattern is not obviously explained by differences in profitability in the different segments of the market.

Our dataset is composed of the price and characteristics for all cars sold as new in Australia in 1998, and registrations (sales) for cars, aggregated by model or make, for Australia in 1998.[4] Both data sets are compiled by a private data-collection firm, Glass's Guide.

[2] One should distinguish between excessive variety for a *given* quality predicted by this paper and larger than optimal product line predicted by standard screening models (e.g. Mussa and Rosen, 1978). These models do not have any horizontal variety, while the vertical size of the product line in these models is the same as in our model.

[3] For reasons similar to those of a monopolist producing a lower than optimal quantity.

[4] Further details on the data are provided in Section 7.

Cars in the Glass's data set are classified by the make and the model, e.g. Toyota is a make and Corolla is a model. Our data set of prices and characteristics contains one observation for each distinct car offered for sale (hereafter called a variety). Though we analyze this data set in more detail in Section 7, for now we compare, in Table 1, averages for three groups of car models, as classified by Glass's Guide, representing three different levels of quality: small, medium and luxury.

The first two columns of the first panel of Table 1 summarize, for each group, the average number of features and average price in Australian dollars.[5] The consistent ranking in terms of number of features and price suggests the ordering of these groups is broadly consistent with ranking by quality. In general, we will consider quality in terms of the cars characteristics, such as the size, fuel efficiency, safety and range and type of features, so to be able to make comparisons across different types of cars. Though specific sets of consumers may have strong preferences for specific sets of features (like smaller cars for easier inner city parking), in general it appears that quality, as reflected by consumers' willingness to pay, is highly correlated with these measures.

The third column, reporting the average number of registrations (sales) by group demonstrates that luxury car sales are less than 10% of total sales and just one fifth those of the other two groups. It is important to keep in mind when considering alternative explanations for flavor proliferation we document more extensively in section 7 whether they are likely to generate so many varieties for such a small number of sales.

The last three columns report the number of makes, models and varieties for each class. Note that the luxury class features the largest number of makes and models. It also makes up 40% of the varieties for new cars, despite making up less than 10% of registrations.

The second panel of Table 1 confirms what is suggested in the first panel. The first three columns report the number of registrations per make, model and variety. These numbers are consistently lowest for the luxury class. The last two columns report the number of

[5]Features are the number of options included as standard with the car, e.g. airbags, stereo system.

models and varieties per make. The number of models per make for luxury cars is more than twice the number for medium cars and more than 1.5 the number for small cars. This is consistent with our focus on a positive relationship between quality and variety. The number of varieties per make is greater for the medium and small cars than for luxury cars though. This could either reflect thinner demand at higher prices or greater numbers of cars being made to order relative to the other classes. That being said, it is still striking that when so few luxury cars are sold that the numbers of varieties per make is still comparable with those for the medium and small classes.

In Table 2, we demonstrate that these patterns occur not only across firms but within firm product lines. We select all firms with models included in at least two of the three groups. Then, for each group, for each firm, we calculate the average price, the average number of features and the average number of varieties per registration. We then calculate the ratio of the average for medium compared with small, luxury compared with medium and, for a few makes that do not offer medium models, the ratio of luxury compared with small. Finally we take the average of the ratios across the firms and report these averages in Table 2. For example, for the nine makes that offer both medium and small cars, on average, medium cars are 46% more expensive and have 69% more features than small cars. And for each car sold there are 21% more varieties per medium car than per small car sold.

In general, the pattern of prices and features across these groups are consistent with those in Table 1. But Table 2 also demonstrates flavor proliferation by firms. First, note that on average, the ratio of the varieties per registration for higher quality to lower quality models for luxury and medium models is greater than one. In other words, luxury and medium models have, on average, more varieties per sale than lower quality models offered by the same firm. Strikingly, there are far more varieties per registration for luxury cars relative to medium cars than luxury cars relative to small cars if the firm does not offer any medium cars. This is consistent with flavor proliferation to price discriminate for a firm would not need to offer as many varieties if the other cars it offers are more distant substitutes.

The simplest explanation for flavor proliferation is that firms offer more varieties of more profitable cars. Hence, more varieties of luxury cars are offered than medium cars because these cars are more profitable. Specifically, profit margins on luxury cars will be greater than other cars if their buyers are less price sensitive as they are more interested in getting a car with the right combination of characteristics. A related argument is that purchasers of higher quality cars may also have more distinctive preferences that support more varieties. This is supported by the estimated profit margins on different models of cars reported by Berry et al. (1995). These largely increase with price, with the most expensive model they report, the BMW 735i, also having a much larger markup than the others.

However, profitability is determined by sales as well as the markup and, as Table 1, demonstrates, the sales of luxury models are just one fifth the sales of the small and medium groups of cars. It seems unlikely that there is not similar distinctiveness of tastes among the 45% of consumers that each purchase small and medium cars than the 10% purchasing luxury cars. The estimates of Berry et al (1995) also do not support luxury cars being more profitable, despite their higher profit margins. Amongst, the models they report, variable profit does not generally simply increase with price. In fact, the BMW 735i has lower estimated variable profits than most of the medium and small cars they consider. And this does not include development costs, which are likely to be much higher for higher quality cars, which would further reduce their profitability. In general, it is not immediately obvious that the profit margins are sufficiently high on luxury cars to overcome much smaller sales and greater development costs, so to make them sufficiently profitable to explain the relatively large number of varieties relative to lower quality cars.

3 Consumer Behavior

In this section we develop a probabilistic choice model of consumer behavior to analyze the implications of introducing new varieties of a differentiated product. The seminal book of Anderson, de Palma and Thisse (1992) summarizes four types of models that lead to probabilistic choice. Drawing on their work, we argue, in an online appendix, that under reasonable conditions, one can move freely between these different interpretations of probabilistic choice models for a fixed set of alternatives. The most important difference between the models, for our purposes, is the simplicity of modelling the change in probabilities if new alternatives are introduced. As we are agnostic about the source of probabilistic choice, for expositional simplicity we use the Luce model, also known as the logit model, which provides the simplest framework incorporating this feature. The first advantage of the Luce model is that choices between varieties depend solely on the utilities associated with each variety. The second feature, and the main property that drives our results, is that adding new varieties leads to a *strict* decrease in the probabilities of choosing all previously available varieties.

The logit model has been extensively criticised on theoretical and empirical grounds for this feature. However, the properties of the logit model we utilize are only a little stronger than the one that necessarily arises in random-utility models, which requires that the probabilities of existing varieties are non-increasing if new varieties are added. Finer properties of the model, which are the focus of recent critiques, are not important for our results. For empirical work, several alternative approaches have been proposed at least some of which are motivated by allowing for unobserved product or consumer heterogeneity. In the theoretical analysis, we simplify things by ignoring such heterogeneity. Furthermore as we do not directly estimate the theoretical model but use it to motivate some tests of its implications, if such heterogeneity is important in our application then it just biases against finding evidence in favour of our model. After reviewing this model in the first subsection, in the second subsection we introduce a concept of near-deterministic behavior, that specifies the very small degree of randomness in consumer behavior required for our results.

3.1 The Luce model

In this subsection we present a simple probabilistic choice model, Luce's (1959) logit model, that incorporates the main property required for our results. We stress that the details of the probabilistic choice model used are not important for our results. The property required for our main theoretical results is that adding a new alternative strictly decreases the choice probabilities of all previously accessible alternatives. The Luce model incorporates this feature in a particularly clear way. Furthermore, it has the simplifying feature that the choice probabilities depend solely on the utilities of the different alternatives. The choice probabilities have the following form:

$$p_i = \frac{\exp(u_i/\lambda)}{\sum_{j=1}^{n} \exp(u_j/\lambda)}. \tag{1}$$

Note that the Luce model implies any two alternatives with the same utility are selected with the same probability. Assuming identical probabilities makes the subsequent analysis relatively simple. Relaxing this assumption only changes the results quantitatively, not qualitatively. In this model parameter λ, which can take values from zero to infinity, can be given several different interpretations. For example, it may reflect the degree of randomness in preferences due to bounded rationality or random fluctuations in preferences. Alternatively, it can also represent the degree of horizontal differentiation of tastes. If $\lambda \to 0$ then

$$\lim_{\lambda \to 0} p_i = \begin{cases} 1/k, \text{ if } u_i = \max\{u_1, ...u_n\} \\ 0, \text{ otherwise} \end{cases}, \tag{2}$$

where integer k is the cardinality of the set of the utility maximizers.[6] Note that this is the first sense in which consumers randomize and that the probability of each choice falls as the number of optimal varieties increases. It is also important to note that this is the main feature of the Luce model that we use for our results. Other features of the model, which have been recently criticized, are not essential for our results.

If λ is greater than zero, then the probability of the consumer choosing a product other than the one that maximizes the non-random or vertical component of utility is positive. This is the second sense in which the consumers randomize across products. For $\lambda < \infty$ the probability of choosing a product increases with the utility provided. At the extreme

[6]Since a fully deterministic consumer or a consumer that does not have any horizontal preferences (corresponding to the case $\lambda = 0$) can randomize with any probability among the utility maximizing choices, the choice correspondence of equation (1) is *not* lower hemicontinuous at $\lambda = 0$.

case where $\lambda \to \infty$ the choice probabilities converge to $1/n$, i.e. the choice becomes totally random and independent of the utility level. Again, the probability of choosing each product falls as the number of varieties increases. Note that the logit formulation is not required for λ to play this role; for example, a probit formulation is also feasible.

3.2 A concept of a nearly deterministic consumer

In this subsection we introduce the concept of a nearly deterministic consumer — which states that the randomness in consumer choice required to yield our results is relatively small, corresponding to a small value of λ. Assume that the probability of different choices by the consumer is given by a continuously differentiable function:

$$p(\cdot) : R^n \times R_+ \to \Delta^n, \tag{3}$$

such that $u_i = u_j$ implies $p_i(u, \lambda) = p_j(u, \lambda)$ and $p(u, 0)$ is given by equation (2). Luce probabilities, as in equation (1), satisfy these properties though the exact form of function $p(\cdot, \lambda)$ is not important for our purposes.

Let M be the set of the utility maximizers, i.e.

$$M = \{u_i : u_i = \max\{u_1, ...u_n\}\}. \tag{4}$$

Take any $u_j \in M$ and define

$$\Delta = \min_{u_k \notin M}(u_j - u_k). \tag{5}$$

Definition 1 *An economic agent whose choice probabilities are given by equation (3) is called nearly* deterministic *if* $\lambda << \Delta$.

In words, the definition says that an economic agent is nearly deterministic if λ is *much smaller* (sign $<<$ reads "much smaller") than the difference in utility between the optimal and the next to the optimal choice. The exact meaning of "much smaller" depends on the extent of randomness to be permitted in decision-making.

Assume that consumers differ in their preferences for quality, i.e. are different types. A standard reaction of the seller to consumer heterogeneity is to devise a product line, where each product will be targeted for a certain type of consumer. In the standard model of price discrimination with fully deterministic consumers infinitely small price changes can be used to get each type of consumer to purchase the product designed for them. Introducing $\lambda > 0$ mutes the effect of price changes on the probability of choice, but we require only a small degree of randomness to obtain our results.

4 The monopolistic screening model

Let us briefly review the basic screening model with fully deterministic consumers[7] and discuss the consequences of offering the second best contract to nearly deterministic consumers. Assume a risk neutral monopolist produces a unit of good with quality x at a cost $C(x)$, where $C(\cdot)$ is a strictly convex, twice differentiable function. Preferences of a consumer, of type θ, over a unit of good with quality x are given by a twice continuously differentiable utility function $u(\theta, x)$. Preferences of the consumer are quasilinear in money:

$$v(\theta, x, m) = u(\theta, x) + m.$$

Each consumer wants to buy at most one unit of the monopolist's goods. Type θ is private information of the consumer. If the consumer does not purchase a good from the monopolist, she receives utility $u_0(\theta)$. For simplicity assume it does not depend on type and normalize it to be zero. Finally, assume

$$u_1 > 0,\ u_2 > 0,\ u_{12} > 0.$$

(Here u_i is the derivative of u w. r. t. the i^{th} argument, u_{12} is the cross partial derivative with respect to θ and x). The last of these conditions is known as the Spence-Mirrlees condition or the single-crossing property.

Let us assume that $\theta \in \{\theta_L, \theta_H\}$. Then the optimal qualities x_L and x_H are characterized

[7] See Mas-Colell, Whinston and Green (1995), Section 13D, for details.

by:

$$\begin{array}{c} u_1(x_H, \theta_H) = C'(x_H) \\ u_1(x_L, \theta_L) - C'(x_L) = \frac{p_H}{1-p_H}(u_1(x_L, \theta_H) - u_1(x_L, \theta_L)) > 0 \end{array} \tag{6}$$

$$\begin{array}{c} t_L = u(x_L, \theta_L) \\ t_H = u(x_H, \theta_L) - u(x_L, \theta_H) + u(x_L, \theta_L) \end{array} \tag{7}$$

Note that x_H is at the efficient level (no distortions at the top) and x_L is below the efficient level. At these prices high value consumers are indifferent between the two products, and low value consumers are indifferent between purchasing the low quality product and not purchasing at all.

To place a bound on λ requires, according to equation (5), considering the gaps in utility between the most preferred and next preferred products for each type of consumer. For type θ_H, the gap in utility between its preferred and next preferred option, equal to their information rent, is:

$$I_{21} = u(x_H, \theta_H) - t_H = u(x_L, \theta_H) - t_L = u(x_L, \theta_H) - u(x_L, \theta_L).$$

For type L, who is indifferent between purchasing x_L and not purchasing, the gap in utility to its next preferred option is:

$$\Delta_{IC} = t_H - u(x_H, \theta_L), \tag{8}$$

which also measures the slack in the incentive compatibility condition for the low type. Hence we can determine

$$\Delta = \min(I_{21}, \Delta_{IC}). \tag{9}$$

From now on, we will assume that

$$u(\theta, x) = \theta x. \tag{10}$$

This specification of the incentive compatibility constraint for the high type together with the condition $\theta_H > \theta_L$ implies a convex relationship between price and quality:

$$t_H - t_L > \theta_L(x_H - x_L). \tag{11}$$

In other words, this implies for a price discriminating firm, that prices increase more than proportionately with quality. This is a special case of a general result that states that the optimal tariff is convex for an arbitrary type space. Indeed, let $\theta \in \Omega \subset R$, where Ω may be a finite set, as in this paper, or even a more general subset of R. Given an implementable allocation $x(\theta)$, which may but need not be optimal, define consumer's surplus by:

$$s(\theta) = \int_{\theta_L}^{\theta} x(\beta) d\beta. \tag{12}$$

This allocation is implementable by tariff [8]

$$t(x) = \max_{\theta}(\theta x - s(\theta)). \tag{13}$$

Equation (13) implies that the tariff implementing $x(\cdot)$ is convex. If Ω is a finite set, the convexity requirement is reduced to:

$$t_{i+1} - t_i \geq \theta_i(x_{i+1} - x_i), \tag{14}$$

where the inequality is strict as long as $x_{i+1} > 0$.

Let us call the contract (6)-(7) the *second best contract*. Assume that the monopolist offers the second best contract to nearly deterministic consumers. By the definition of a nearly deterministic consumer, parameter λ is much less than both the high type information rent and the slack in the low type incentive compatibility constraint. Therefore, the fraction of high type consumers who decide not to participate or the fraction of the low type consumers who decide to choose the high quality product is negligibly small.

On the other hand, as a result of randomizing between equally preferred alternatives, similar to that described in equation (2), approximately half of the low type consumers decide to stay out of the market and approximately half of the high type consumers purchase the low quality product. This leads to a drop in the monopolist's profits which is higher order of magnitude than λ. If consumers were fully deterministic, then infinitely small price

[8] See, for example, Basov (2005) for a discussion of implementability in general screening models.

changes deal with this problem but, as we have argued, infinitely small price changes will not be sufficient if consumers are nearly deterministic. Instead the monopolist must alter prices to violate the binding constraints by some finite amount.[9] This reduces the profits earned from both high and low types.

The alternative to a significant price cut is for the monopolist to create multiple flavors for each quality level. To see how multiple flavors help, assume the monopolist sells m high quality flavors and one low quality flavor. Now high type consumers are faced with $(m+1)$ choices, each of which provides them with the same utility. The probability that the high type consumer purchases the high quality product is $m/(m+1)$. If the marginal cost of adding a new flavor is sufficiently low this way of ensuring participation may be preferable to leaving extra rents to the consumers.[10] From the social point of view, flavor proliferation is likely to be excessive — particularly if there is no horizontal component of preferences. Next we investigate flavor proliferation in more detail.

5 The flavor proliferation model

In this section we analyze how flavor proliferation increases the profitability of price discrimination by overcoming the problem of consumers randomizing away from the most profitable product for their type. We demonstrate, for the two type case, that the number of flavors strongly increases with product quality. If the cost of adding a new flavor converges to zero the ratio of the numbers of flavors for adjacent quality levels converges to infinity.

If the monopolist offers n flavors of low quality and m flavors of high quality, the low quality good will be purchased by a fraction q_L of the consumers, where

$$q_L = (1 - p_H)\frac{n}{n+1} + p_H \frac{n}{n+m}, \tag{15}$$

[9]Basov(2009) shows that the required change in tariffs in an optimal contract is of the order of $\lambda/\log\lambda$, which corresponds to the probability of making a mistake of order λ.

[10]Our assumption that only vertical differences determine preferences over products except for a random element due to horizontal preferences or other sources of randomness has some empirical support. For example, Draganska and Jain (2005a) report preferences for quality are much stronger than those for flavor.

while the high quality good will be purchased by a fraction q_H of the consumers, where

$$q_H = p_H \frac{m}{n+m}. \tag{16}$$

Before proceeding, we note three assumptions. First, we assume that the marginal cost of adding a new flavor is $c > 0$ and does not depend on the quality. Second, we assume the *vertical* utility for each type is the product of the quality of the good consumed and the type, θ. Finally, under conditions specified at equation (29), we can use the optimal qualities for deterministic consumers as an approximation for those optimal for nearly deterministic consumers. Therefore, the monopolist solves:

$$\max_{m,n}((t_H - C(x_H))q_H + (t_L - C(x_L))q_L - c(n+m)). \tag{17}$$

Let us introduce the following notation:

$$\pi_L = t_L - C(x_L),\ \pi_H = t_H - C(x_H),\ p_L = 1 - p_H, \tag{18}$$

i.e. π_i are the profits per consumer the monopolist can potentially earn on type i if all consumers of this type select the contract designed for them. Note that $\pi_H - \pi_L > 0$. Indeed,

$$\pi_H - \pi_L = t_H - t_L - C(x_H) + C(x_L). \tag{19}$$

Using expressions for the tariffs, based on our assumed utility function:

$$\pi_H - \pi_L = \theta_H(x_H - x_L) - C(x_H) + C(x_L) + \theta_L x_L. \tag{20}$$

Finally, since the optimal quality is efficient for the top type, $C'(x_H) = \theta_H$ and strict convexity of the cost implies:

$$\pi_H - \pi_L = C'(x_H)(x_H - x_L) - C(x_H) + C(x_L) + \theta_L x_L > 0. \tag{21}$$

The monopolist's problem can be rewritten as:

$$\max_{m,n}(p_L \pi_L \frac{n}{n+1} + p_H \pi_H \frac{m}{n+m} + p_H \pi_L \frac{n}{n+m} - c(n+m)). \tag{22}$$

Ignoring the constraint that m and n should be integers, the first order conditions are:

$$\begin{cases} \frac{p_L\pi_L}{(n+1)^2} - \frac{p_H(\pi_H-\pi_L)m}{(n+m)^2} = c \\ \frac{p_H(\pi_H-\pi_L)n}{(n+m)^2} = c \end{cases} . \tag{23}$$

It is easy to observe that for small values of c

$$n = \frac{p_L^{2/3}\pi_L^{2/3}}{p_H^{1/3}(\pi_H-\pi_L)^{1/3}c^{1/3}} + O(c^{1/3}) \tag{24}$$

$$m = \frac{p_L^{1/3}p_H^{1/3}\pi_L^{1/3}(\pi_H-\pi_L)^{1/3}}{c^{2/3}} + O(c^{1/3}). \tag{25}$$

As $c \to 0$ both n and m go to infinity, but in a such way that

$$\frac{m}{n^2} \to \frac{p_H(\pi_H-\pi_L)}{p_L\pi_L}. \tag{26}$$

As long as the profits from the high quality product are high enough, and the probability of the consumer being a high type is not too low, m/n increases proportionally with n. Finally, flavor proliferation costs the monopolist:

$$F = c^{1/3}(\pi_H-\pi_L)^{1/3}\pi_L^{1/3}p_H^{1/3}p_L^{1/3}. \tag{27}$$

Basov (2009) demonstrates that a monopolist who faces nearly deterministic consumers, as defined in Definition 1, and is restricted to offering a binary menu (i.e. no flavor proliferation is possible) will find it approximately optimal to leave qualities at the same level as for the deterministic consumer and to alter prices.[11] However, if flavor proliferation is allowed and

$$F << \lambda \tag{28}$$

holds, then it is even more profitable to proliferate flavors than to alter prices. Hence, under the following conditions, flavor proliferation, at the optimal qualities and prices derived earlier is approximately optimal:

$$F << \lambda << \Delta, \tag{29}$$

[11] Basov (2009) demonstrates the difference in the optimal qualities for consumers that are deterministic and consumers that are nearly deterministic is at most of order $(\frac{\lambda}{\log\lambda})^2$ which, under these conditions, is very small. The differences in optimal tariffs in the two cases is also very small being $O(\frac{\lambda}{\log\lambda})$. Hence, the optimal solution for deterministic consumers can be used without materially altering the results.

where Δ is defined by (9).

Note that if c is sufficiently small both m and n are large, moreover, m/n is large. Therefore, the consumers will choose the options designed for them with probabilities close to one, as predicted by the screening model with fully deterministic consumers.

6 An extension of the model: multiple types.

So far we have discussed the monopolistic screening model with two types of consumers. We argued that if consumers are not fully deterministic, the monopolist might be better off offering multiple flavors of a high quality product, even if consumers care only about quality. In this section we extend the model to include more than two types of consumers. In the first subsection we demonstrate the number of varieties increases faster than exponentially if per-consumer profits increase sufficiently fast with the quality level and markets for high quality varieties are sufficiently thick. In the second subsection, we demonstrate the relationship between quality and variety is hump-shaped if markets are thinner at higher prices.

The assumptions on the fundamentals are the same as in Section 4, but now the consumer's type is given by $\theta \in \{\theta_1, ..., \theta_N\}$. Let $p_i = \Pr(\theta = \theta_i)$. Otherwise the analysis in this subsection proceeds in the same way as in Section 5. We assume $p_i > 0$ for all i and

$$\sum_{i=1}^{n} p_i = 1. \tag{30}$$

Denote by (x_i, t_i) the quality and tariff the monopolist would have offered to type θ_i had the consumers been fully deterministic. For $i = 1, ..., N$ define π_i by:

$$\pi_i = t_i - C(x_i) \tag{31}$$

and let $\pi_0 = 0$ be the profit the monopolist earns from the consumers who choose the outside option. Finally, define $\Delta\pi_i = \pi_i - \pi_{i-1}$. First, note that $\Delta\pi_i > 0$ for all i. Indeed,

$$\Delta\pi_i = t_i - t_{i-1} - C(x_i) + C(x_{i-1}). \tag{32}$$

The constraint reduction theorem (Stole, 2000) implies that the only binding constraints in the problem are the individual rationality constraint for the lowest type and the incentive compatibility constraint between types θ_i and θ_{i-1}. Hence, we can exclude the tariffs to get:

$$\Delta\pi_i = \theta_i(x_i - x_{i-1}) - C(x_i) + C(x_{i-1}) + \theta_{i-1}x_{i-1}. \tag{33}$$

Finally, since the optimal quality is efficient for the top type and biased downward for the rest, $C'(x_i) \le \theta_i$. Strict convexity of the cost implies:

$$\Delta\pi_i \ge C'(x_i)(x_i - x_{i-1}) - C(x_i) + C(x_{i-1}) + \theta_{i-1}x_{i-1} > 0. \tag{34}$$

Using the same approximation as in the two types case, the monopolist's problem can be written as:

$$\max_{\{n_i\}_{i=1}^N} \sum_{i=1}^{N} (p_i\pi_i \frac{n_i}{n_i + n_{i-1}} + p_i\pi_{i-1}\frac{n_{i-1}}{n_i + n_{i-1}} - cn_i), \tag{35}$$

where we defined $n_0 = 1$. Ignoring the constraint that n_i should be an integer, one can write the first order conditions:

$$\begin{cases} \frac{p_i\pi_i}{(n_i+n_{i-1})^2} - \frac{p_{i+1}\Delta\pi_i n_{i+1}}{(n_i+n_{i+1})^2} = c \\ \frac{p_i\Delta\pi_i n_i}{(n_i+n_{i+1})^2} = c \end{cases}. \tag{36}$$

As $c \to 0$ all n_i go to infinity, but in a such way that

$$\frac{n_{i+1}}{n_i^2} = \frac{p_{i+1}\Delta\pi_{i+1}}{p_i\Delta\pi_i}. \tag{37}$$

Behavior of number of flavors with quality depends crucially on parameter $p_{i+1}\Delta\pi_{i+1}/p_i\Delta\pi_i$. If this parameter is greater than one for all i, than number of flavors increases fast with the quality rank. The value of n_1 is given by:

$$n_1 = \frac{(p_1\pi_1)^{2/(2N-1)}}{(p_2\Delta\pi_1 c)^{1/(2N-1)}} + O(c^{1/(2N-1)}) \tag{38}$$

and n_i for $i > 1$ can be calculated using (37). Finally, the flavor proliferation costs of the monopolist are:

$$F_N = c^{(N-1)/(2N-1)} \prod_{i-1}^{N} (\Delta\pi_i p_i)^{1/(2N-1)} \tag{39}$$

Once again, flavor proliferation is approximately optimal as long as:

$$F_N << \lambda << \Delta, \tag{40}$$

where Δ is defined by the maximal "slack" for the non-binding constraints (in the case of two types given by equation (9)). As one can see from equation (37), the number of flavors increase with quality at an increasing rate, provided that p_i and p_{i+1} are of the same other of magnitude, i.e. in that case the increase of number of flavors with the quality rank is faster than exponential. This represents a wasteful proliferation of flavors, though if one assumes that flavors are directly valued, this may offset some of this effect. If we instead assume that

$$\frac{p_{i+1}\Delta\pi_{i+1}}{p_i\Delta\pi_i} < 1 \tag{41}$$

for all i the relationship between the number of varieties and quality is hump shaped, increasing and then decreasing, rather than monotonically increasing over the whole range.

There are several reasons why an assumption like equation (41) may hold which, for convenience, we will discuss in terms of our empirical application, cars. The most obvious cause is that consumers with strong preferences for quality (or snobs) are sufficiently rare.

There is a second set of reasons for why individual companies may offer a hump shaped set of varieties. If the company has been most successful producing cars of a certain quality, consumers may be unlikely to immediately accept models claimed to be higher quality and require a lower price, compared with the models sold by established high-quality producers. This makes producing varieties outside of the range of quality currently accepted by consumers less profitable. This argument doesn't explain, though, why the same company cannot successfully introduce varieties of a lower quality than their most successful models.

An alternative explanation on the cost side is that if the firm produces a quality level less than or greater than the quality level at which the number of brands is greatest, production costs may be higher. Assume each car-maker can produce a particular quality of cars very cheaply but that there is a U-shaped average cost curve (as a function of quality). Hence, if the firm produces either higher or lower quality cars average costs are higher relative to the price that can be obtained for the car and profits are relatively small from producing them. Hence, the firm offers less varieties at these quality levels.

Both of these hypotheses are difficult to test without considerable data. However, the frequent rebadging of, particularly smaller, cars for sales in different markets suggests that production costs are unlikely to be the whole story. It may be too expensive for Mercedes to make a cheap small model in the same factories that make Mercedes, but they could license Daewoo to do so at the Daewoo factories.

To formally analyze the implications of equation (37) we introduce the following notation:

$$\xi_i = \ln n_i \tag{42}$$

$$\beta_i = \frac{1}{2} \ln \frac{p_i \Delta\pi_i}{p_{i+1}\Delta\pi_{i+1}} > 0 \tag{43}$$

$$\beta(x) = \beta_i \text{ for } x \in (i-1, i]. \tag{44}$$

Equation (37) now takes the form

$$\xi_{i+1} - 2\xi_i = \beta_i \tag{45}$$

and its solution can be written as:

$$\xi(x) = \xi_0 2^x - \int_0^x \beta(y) 2^{x-y} dy. \tag{46}$$

We also assume that $\xi_0 > \beta_0$ (this assumption means that the logarithm of the flavors at the lowest quality level is sufficiently big, which is consistent with our assumption of the small cost of flavor proliferation). To find maximum of $\xi(\cdot)$ w.r.t. x, note that

$$\xi'(x) = 2^x \ln 2(\xi_0 - \frac{\beta(x)2^{-x}}{\ln 2} - \int_0^y \beta(y)2^{-y} dy). \tag{47}$$

Therefore,

$$[\xi'(x) = 0] \Leftrightarrow [\xi_0 = \frac{\beta(n)2^{-n}}{\ln 2} + \sum_{i=0}^{n} 2^{-i}\beta_i]. \tag{48}$$

Since the right hand side of this expression increases in n there is at most one n for which this condition is satisfied. Hence, the relationship between quality and variety is hump shaped, rather than increasing over all qualities.

Note that our model predicts *an* excessive variety for a given quality, which should be distinguished from larger than optimal product line predicted by standard screening models (e.g. Mussa and Rosen, 1978). These models do not have any horizontal variety, while the vertical size of the product line in these models is the same as in our model.[12]

7 An empirical analysis of car varieties

In this section we analyze if flavor proliferation is being used to price discriminate in the Australian car market. After presenting the data to be used, we analyze the shape of the density of flavors offered by each substantial make in the Australian car market. We determine that the typical density is unimodal or has one mode much larger than the other. Nearly all densities are positively skewed. In the next subsection, we summarize a recently proposed model of product differentiation, highlighting the important role of demand. In the final subsection, using a set of Kolmogorov-Smirnov tests, we test if the distribution of flavors matches the distribution of demand or is consistent with the flavor proliferation hypothesis. We demonstrate for most makes that there are relatively more varieties at higher prices consistent with flavor proliferation for higher quality models offered by firms.

7.1 Data

The data we use for this analysis is the same data set, originally collected and compiled by the private data-collection firm Glass's Guide, used in Prentice and Yin (2004). The first component of the data set contains the prices and characteristics for all varieties of cars sold as new in Australia in 1998. The second component of the data set is registrations, usually

[12] Note also that the technical reason why a small deviation from deterministic choice leads to a significant change in the optimal product line, adding an extra dimension to it, which is not small is the failure of lower hemicontinuity of the choice correspondence in the parameter that captures the degree of indeterminacy λ (see, Basov 2009). This situation is quite general and not restricted to the Luce model.

by model or make, for Australia in 1998. [13]

Although our model focuses on the relationship beween quality and variety, it proved very difficult to obtain a useful measure of quality. While rankings are published in newspapers, car magazines and directories, these tend to have too little variation to be useful. For example, the most comprehensive publicly available set of rankings, for Australia in 1998, are in the Dog and Lemon Directory. It, similar to other ratings, rates models from 1 to 5 stars. However, the vast majority of models receive between 1 and 3 stars and there tends to be much more variation across makes than within makes. Hence, there is insufficient variation for comparing 30 to 80 different varieties within a make. And, as tends to be case in other media ratings, the ratings were compiled within groups rather than across groups. This results both very small cheap cars and prestige cars receiving 5 star ratings. This may be sensible for within group comparisons but in our study we need to be able to make detailed comparisons across varieties across groups within the same firm.

Instead, we use follow the extensive hedonic pricing literature on cars that specifies price as a function of the product characteristics we consider correlated with quality. Prentice and Yin (2004) report ten studies alone, between 1961 and 2004, that focus solely on constructing quality adjusted price indices for cars using this approach. Furthermore between 2004 and 2010, a further nine studies have been published applying the hedonic approach to valuing cars in general. Furthermore, our theoretical model suggests this is a conservative approach that biases against finding a statistically significant increasing relationship between price and the number of varieties. Specifically, as demonstrated in equation (11), there is a convex relationship between quality and price. This implies for a given increase in quality there is a relatively rapid increase in price. Hence, the relationship between price and the number of varieties will be flatter than the relationship between quality and the number of varieties.

[13] For higher priced cars, though, registrations may only be available for groups of models. For example, Mercedes Benz has six different models in the C-Class in 1998, whereas the registrations data is available only for C-Class, hence aggregating over the six models. Similarly, the Glass price data distinguishes between the Commodore and two higher quality versions of the Commodore, the Berlina and Calais, whereas in the registrations data, the Commodore, Berlina and Calais are aggregated as Commodores.

Four makes were manufactured in Australia in 1998 — General Motors-Holden, Ford, Toyota and Mitsubishi — but many other international makes were also sold there. We limit our sample to the seventeen makes offering 30 or more flavors. These makes range from the low price Daewoo and Daihatsu to the premium priced BMW and Mercedes-Benz. We also exclude all flavors with prices of $200,000 or more. This is because we believe that such cars are not generally sold as off-the-shelf varieties, so numbers of flavors will not be informative.

7.2 Density of flavors

The first step is to analyze the density of flavors for each make in our sample. We first analyze if the densities increase over the range of flavors or are hump shaped as described in subsections 6.1 and 6.2. We estimate kernel density functions of the flavors of each make. The graphs of these rule out the first case, so we focus on the second case.

The simplest way to determine if the densities are hump-shaped is to perform a non-parametric dip test.[14] The dip test tests if there is a significant difference between the empirical distribution function and a unimodal distribution calibrated to the data. For eleven of the seventeen makes, we fail to reject unimodality at a 5% significance level. The eleven makes include the relatively cheaper makes like Daewoo and Hyundai, all makes manufacturing models in Australia (though two would have rejected at 10%) and the premium brands Audi and Mercedes Benz. However, the kernel density functions of the other six makes, as shown in Figure One, are broadly hump-shaped. With the exception of Daihatsu, each has a tendency to have more flavors at high prices which is consistent with our model.

Calculation of the coefficient of skewness for each density reveals that nearly all of them are positively skewed. For the seventeen makes, only one make had negative skewness (-0.32), seven more had skewness statistics between 0.08 and 0.45, six more had skewness statistics between 0.5 and 0.9 and three had skewness statistics between 1.82 and 3.52.

The final step we took to determine the typical shape of the densities of flavors was

[14] We use the GAUSS code of Henderson et al (2008) who applied the Cheng and Hall (1998) version of the test.

to estimate a beta density function for each set of flavors.[15] The beta density function is described by two parameters, α and β which can take on a wide variety of shapes depending on their values. Before estimating the densities for each make, we had to transform the data. This is because price is a continuous variable with similar cars selling for similar but not identical prices. We obtained data for estimating the density function in three steps. In the first step we construct a kernel estimate of the density for each make. Second, we select values of the density function at 50 points. To ease estimation, we convert these values to integers *by* multiplying them by 10 million and rounding. Descriptive statistics of the new densities revealed they were similar to the descriptive statistics for the prices. However, we will not be able to perform meaningful hypothesis tests with this new data.

If both parameters have values greater than one, then the distribution is unimodal and hump-shaped. We find this to be the case for thirteen out of the seventeen makes. For one make, BMW, $\alpha = 1$ and $\beta > 1$ which implies the density declines towards 0. The densities of the other three brands are also downward sloping but in a J-shape. These makes tend to be more expensive, including Audi and Saab. However, other premium makes, such as Mercedes Benz and Volvo, were included among the thirteen.

Cheng and Hall (1998) suggest their calibration of the dip test may not be suitable for highly skewed data and, from the results of estimating beta densities, three out of the four makes with non-hump shaped distributions featured highly skewed distributions. Hence, for robustness, we decided to re-perform the tests without the skewing observations. For the nine makes with skewness coefficients of 0.5 or more, we dropped the top 10% of observations. For the five makes with lower skewness after this, we performed the dip-test and re-estimated the beta density functions to determine the influence of skewness on the results. For the dip test, none of the makes we estimated were amongst those for which unimodality had been rejected. The p-values on the test statistic fell for all of them, though not enough to change the result. The results from estimating the beta density function were more striking. One of

[15]This is done using the betafit ado program for Stata.

the five had previously returned coefficients suggesting a J-shape. Now both α and β were above one, and for the other four makes, their coefficients were much closer together. So it seems as though skewness did not affect the dip statistic but it did affect the results from estimating the beta density function.

The results from both a non-parametric test and estimating a density function are consistent with the firms offering a hump shaped distribution of flavors. However, this descriptive analysis doesn't determine whether the pattern results from a strategy of flavor proliferation or is due to another cause. Before doing this, though, we need to explore alternative explanations of the set of flavors offered by firms.

7.3 Alternative hypothesis

The primary alternative explanation of the pattern described in Section two is that firms offer varieties if sufficient numbers of consumers demand each variety so to make it's provision profitable. In other words, the consumers differ substantially not only in their preferences for quality, but also in their preferences for flavor, i.e. λ is sufficiently large.

Draganska, Mazzeo and Seim (2009) (hereafter DMS) provide a model of competition by duopolists who choose price and which flavors of a product to offer that is designed for estimation. Like the few other models for estimation that endogenize quality choices, the set of flavors to be offered is chosen in the first stage of a two stage game. In the second stage, given the set of flavors chosen by each firm, an equilibrium set of prices is chosen for each set of flavors and consumers choose which products they purchase. Hence, the set of flavors each firm chooses to offer is determined by the expected profitability of each flavor in equilibrium.

DMS begin by modelling the second stage of competition given a set of flavors. On the demand side DMS's model obtains a flavor level demand equation by aggregating across discrete choices of flavors by consumers, as developed by Berry et al. (1995). Denote s_{bf} be the market share of flavor f of brand (or firm) b which is a function of the flavor's price, p_{fb} and characteristics, x_{fb}, the prices and characteristics of other flavors offered, p_{-fb}, x_{-fb}, and market demographics, ν:

$$s_{fb} = f(p_{fb}, p_{-fb}, x_{fb}, x_{-fb}, \nu) \tag{49}$$

Total sales will equal the product of the market share and market size. On the supply side, DMS assume firms engage in Bertrand competition with differentiated products. The set of flavors offered for each brand can be represented by a vector of dummy variables, d_b, where each element takes a value 1 if a flavor is offered. Each firm chooses a set of prices to maximize profits from offering all of its flavors. Denote d_{-b} as the set of flavors offered by all of the other firms. The equilibrium set of prices offered by each firm are written as a function of marginal cost, c_{bf} and a markup term. All variable without subscripts denote the set of variables offered by all firms:

$$p_{bf}(d_b, d_{-b}) = c_{bf} + MU_{bf}(X, p, d, c, \nu) \tag{50}$$

Next DMS consider's the first stage of competition. In this stage each firm chooses d_b to maximize expected profits. Denote h_b as the vector of fixed costs of firm b operating each flavor and $\bar{\Pi}(d_b)$ as the expected variable profit from offering d_b. The expected profit from offering d_b is given by:

$$E[\Pi_b(d_b, d_{-b}] = \bar{\Pi}(d_b) - h_b' d_b \tag{51}$$

Hence, the number of flavors offered by a firm at quality level, l, is modeled as:

$$n_{lb} = f(X, h, c, \nu, d_{lb}) \tag{52}$$

Production and entry costs are typically estimated rather than observable in most econometric models of competition between differentiated products. The car industry is a lot more complicated than the market analyzed by DMS, so it is not feasible to get estimates of these by estimating a model like DMS's. Furthermore, data on demographics specific to different quality levels tends to be limited as well. Finally, explicitly modeling the relationship between the number of flavors offered by a firm and all other flavors offered is not straightforward. Hence, it is infeasible to estimate even a reduced form model to test whether flavor proliferation rather than demand and supply side factors determine the pattern of varieties.

Furthermore, there may be multiple sets of specifications of preferences and costs that could generate a positive or hump shaped relationship between quality and variety. For example, the variety of preferences could increase with income thereby supporting more high quality variants in equilibrium. Alternatively, a hump-shaped distribution of varieties could occur if a firm has low marginal costs (or low fixed costs) of producing a certain quality level but higher costs for producing at any other level of quality. So using dummy variables to control for many unobservable is unlikely to be successful. This model does not produce a clear cut conclusion for the variety of flavors. In principle, one may expect distributions of different modalities for different markets. The fact that the shape of the distribution is similar across makes provides an indirect support for our theory.

7.4 A test of the hypothesis

The analysis of the previous subsection suggests determining if firms are engaging in flavor proliferation by a reduced form analysis of the number of varieties by quality level by make is unlikely to be feasible or successful. Instead, we propose a test of an implication of our model that can be tested without relying on detailed controls for the determinants of numbers of variants across distinct quality levels and segments of the market.

Specifically, whether the number of flavors offered by a firm increases with quality or whether the number of flavors has a hump shaped relationship with quality, in both cases, we expect a greater share of flavors offered by a firm to be associated with higher quality models. While such a relationship could occur as a result of costs and preferences, our analysis in Section 2 suggests this is unlikely.

We construct the test as follows. First, for each firm, we rank all varieties by price. The null hypothesis is that the distribution of varieties over price, denoted $n(p)$, is determined by their profitability. While we don't observe profitability the sales of all varieties across all firms at that price, denoted $S(p)$, is a good proxy. Sales are likely to be low for high quality prestige cars because their development and production costs are so high as to enable them only to be offered, in equilibrium, at a high price. The mark-up may be high but total profits are unlikely to be too large. Sales are likely to also be low for very low quality small cars because, in part, they have relatively poor characteristics and options and they face competition from second hand higher quality cars. Though low quality cars can be produced at a low cost and sold at a low price total profitability is unlikely to be high.

Hence, under the null hypothesis, the distribution of flavors at each price will not be significantly different from the distribution of total demand at each price.

However, if our model is correct, then the distribution of sales should be to the left of the distribution of flavors. At low quality levels, there will be relatively less flavors, compared with demand, whereas at high quality levels there will be relatively more flavors compared with demand. Formally:

$$F_{S(p)}(z) \geq F_{n(p)}(z) \tag{53}$$

We specify $\geq$ for, as the distributions are compiled over the same range of prices, they must ultimately meet.

To carry out this test, we perform two one-sided Kolmogorov-Smirnov tests of the equality of distributions. Under the null hypothesis, the two distributions are identical. In the first test, we test against the alternative that:

$$F_{S(p)}(z) \geq F_{n(p)}(z) \tag{54}$$

In the second test, we test against the alternative that:

$$F_{S(p)}(z) \leq F_{n(p)}(z) \tag{55}$$

If we reject the null hypothesis in the first test and fail to reject in the second, we conclude that the distribution of varieties is consistent with our model for at least some range of quality levels. Note, two tests are required for as the Kolmogorov-Smirnov test selects points along the distribution it is possible that the distributions may cross and one distribution may be significantly greater than and less than the other at different prices.

It is worth noting two measurement issues. First, we do not observe sales by variety but by models or even groups of models. We estimate sales by variety by assuming each variety has an equal share of the sales for the model. This may underestimate (and is unlikely to systematically overestimate) sales for cheaper varieties within a model, which biases against finding support for our model.

Second, as noted earlier, by not including sales of second hand cars, we are underestimating the sales of cars at, particularly, lower prices. Hence, we may find it harder to reject the null hypothesis that the distribution of flavors matches that of sales at lower prices.

7.4.1 Test results

The test results are summarized in Table 3. In this table we report, with the makes ordered by their mean price, the test statistics for the two Kolmogorov-Smirnov tests. First, we note that the distribution of flavors is significantly below that of sales for all makes except for Mazda, Toyota and the three cheapest brands. Furthermore, for all makes which satisfy the first test, the distribution of flavors is never significantly above the distribution of sales. Figures Two and three present the densities and distributions of sales and flavors for the highest and lowest brands in this group - Subaru and Mercedes Benz. For Mercedes Benz the two densities in the top panel demonstrate that the mass of varieties are provided well to the right of mass of total sales across the range of prices at which Mercedes Benz offers flavors. This leads to starkly different distribution functions in the lower panel. For Subaru, which offers flavors priced below $50,000, while the distribution of flavors tracks the distribution of sales at most prices, nevertheless at the higher prices, the distribution of varieties is significantly below before rising rapidly at the very highest prices (quality levels).

Hence, most makes feature a distribution of flavors consistent with our model. Furthermore, the fact that this feature is common to makes from Subaru and Mitsubishi to

Mercedes-Benz and BMW suggests that this pattern is strategic rather than determined by cost or preferences specific to particular segments of the market (like the variety of preferences increases with income).

It is also striking that the makes for which the three makes for which the model is not supported are in the range where the measurement problems with sales are likely to be most acute. Furthermore the densities of flavors offered by these makes are broadly similar to those offered by the other makes. The results of the tests in subsection two reveal Hyundai to have a hump shaped unimodal density of flavors. Figure One demonstrates Daihatsu and Nissan have hump shaped (if not unimodal) densities.

8 Conclusions

In this paper we have analyzed theoretically and empirically the links between quality and variety. The theoretical model has two components. First, consumers are modeled as caring about quality but indifferent between varieties of the same quality. However, we adapt a standard probabilistic choice model to introduce a very small amount of randomness into consumer behaviour. The second component is a monopolist who engages in second degree price discrimination. Faced with consumers which randomize, the profitability of price discrimination is increased by the monopolist offering more varieties at higher qualities as the likelihood that consumers choose high price products increases. If, at high quality levels, the markets become sufficiently thin, though, the profit maximizing number of varieties falls, yielding a hump-shaped relationship between variety and quality.

We then test these predictions using data on the product offerings of seventeen major car companies in the Australian market. First, we demonstrate that most companies offer a density of varieties that is hump shaped with respect to our proxy for quality, price. This is a previously unidentified empirical regularity. We then determine that this distribution is more likely to be due to flavor proliferation than just meeting demand for different varieties. Formally, we demonstrate that the distribution of varieties is statistically significantly different from the distribution of varieties suggested by consumer demand. Another important contribution of the paper is that the number of varieties offered by the market can be higher than the social optimum even in the case of a single monopoly, which is opposite to the conclusions drawn from the previous literature (see, Lancaster (1990)).

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Table 1
VARIETIES OF NEW CARS IN AUSTRALIA, 1998

Category	Features	Price	Reg'ns	Makes	Models	Varieties
All	11	59.6	584.3	37	175	1081
Small	7	22.6	251.4	19	39	397
Medium	11	34.6	268.6	9	16	207
Luxury	15	114.3	47.9	26	97	390
		Registrations per			Models	Varieties
Category		Make	Model	Obs.	Per Make	Per Make
All		15.79	3.33	0.54	4.73	29.22
Small		13.23	6.44	0.63	2.05	20.89
Medium		29.84	16.78	1.30	1.78	23.00
Luxury		1.84	0.49	0.12	3.73	15

Note: Price and Features data supplied by Glass's Guide. Registrations data from Glass's Guide (1998). Registrations is in thousands. Price is in thousand Australian dollars. All includes Small, Medium, Luxury, Sports and People Movers. Small combines the Glass categories of Small and Light. Medium combines the Glass categories of Medium and Upper Medium. Luxury combines the Glass categories of Prestige and Luxury. All cars are grouped by Glass Classification of the Model, or Make if no model detail.

Table 2
VARIETY PROLIFERATION AND VERTICAL DIFFERENTIATION BY FIRMS

Ratio of	Prices	Number of Features	Varieties per Registration	Cases
Medium to Small	1.46	1.69	1.21	9
Luxury to Medium	1.72	1.42	5.13	3
Luxury to Small (no Medium)	2.04	1.56	1.89	5

Each cell in the first three columns is calculated in three steps:
1. Calculate the average across varieties within each class for each firm
2. Take the ratio of the average across classes for each firm
3. Calculate the average ratio across firms
Fourth column reports number of firms which offer products across each class

Table 3
COMPARISON OF DISTRIBUTIONS OF SALES AND FLAVORS

Brand	Tests	
	$F(n((p)) < F(S(p))$	$F(n(p)) > F(S(p))$
Daihatsu	-0.0319	0.3116***
Daewoo	-0.1151	0.1372
Hyundai	-0.0184	0.2781***
Nissan	-0.0554	0.4035***
Toyota	-0.078	0.075
Subaru	-0.1507**	0.0193
Mazda	-0.1041	0.0793
Mitsubishi	-0.1579*	0.0991
Ford	-0.1778***	0.0166
GM-Holden	-0.219***	0.0046
Honda	-0.3292***	0.0614
Volkswagen	-0.3620***	0.0295
Saab	-0.4773***	0.0278
Volvo	-0.4370***	0.0222
Audi	-0.574***	0.0263
BMW	-0.5684***	0.0174
Mercedes Benz	-0.7598***	0.0163

Note: Price data supplied by Glass's Guide. Registrations data from Glass's Guide (1998).
Note: Significance levels: 1% ****, 5% **, 10% *.

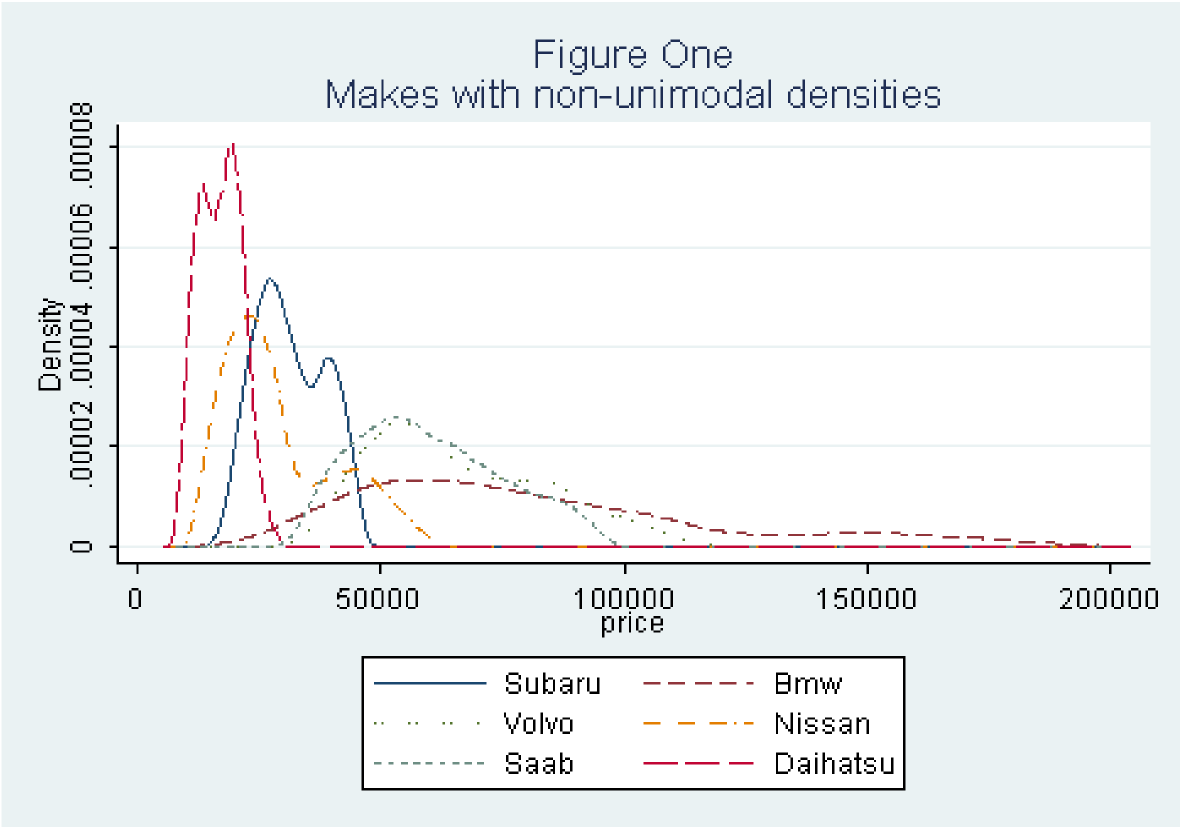


Figure 1: Caption for figone

Figure Two
Density and distribution of sales and varieties of Subaru

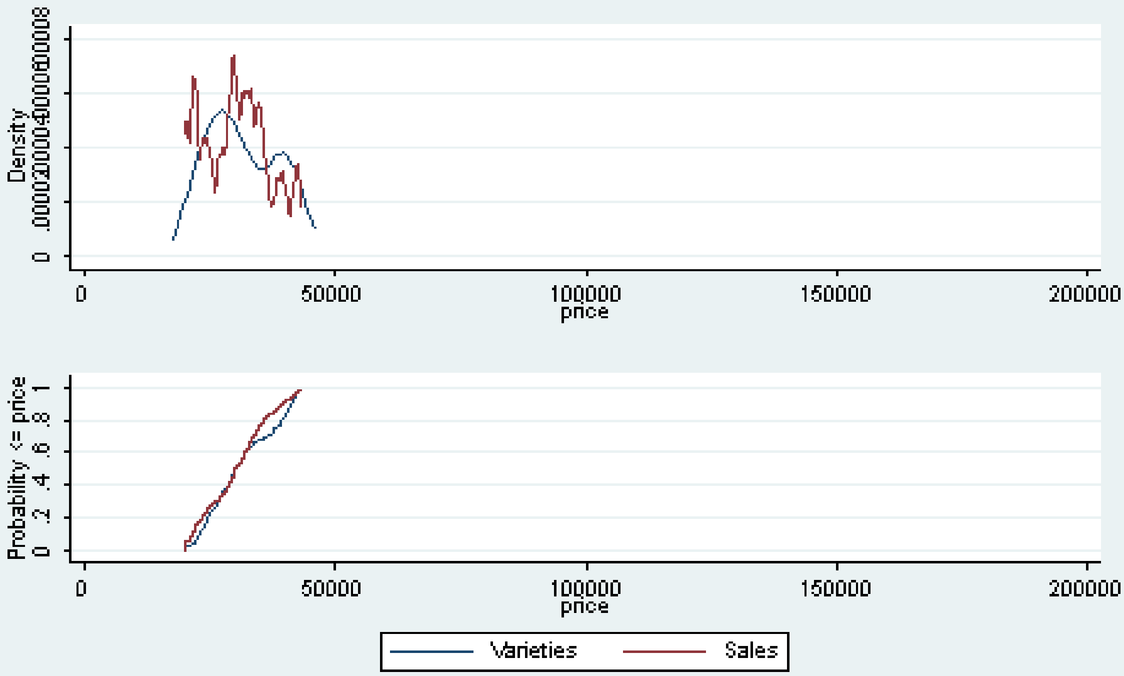
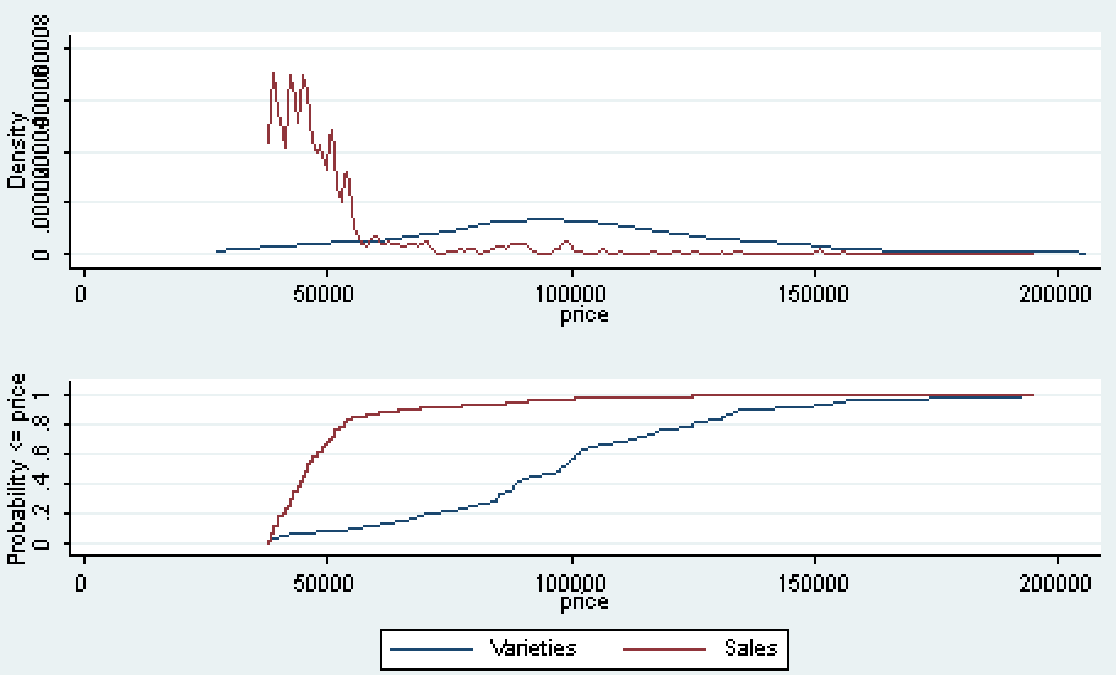


Figure Three
Density and distribution of sales and varieties
of Mercedes Benz



Appendix

In this Appendix, we draw on Anderson et al (1992) and Fudenberg and Levine (1998) to compare four models of probabilistic choice. This comparison demonstrates two points. First, under some reasonable assumptions, one can move freely between different interpretations of random choice models. Second, that the Markovian and Machina-type models provide an easier framework than the random-utility and address models, to analyse the effect of introducing new varieties. Hence in the body of the paper we use the Luce model of probabilistic choice as a particularly simple foundation for our model of consumer behavior. Though one could use a random-utility or address model, modelling would just be more complex.

Markovian learning models

Though these models originated in mathematical psychology in the work of Bush and Mosteller (1955), they have been widely used in economics (e.g. Foster and Young (1990), Fudenberg and Harris (1992), Kandori, Mailath, Rob (1993), Young (1993), Friedman and Yellin (1997), Anderson, Goeree, and Holt (2004), Friedman (2000), and Basov (2003)). Though most economic applications assume a continuous choice space, for simplicity of presentation and for consistency with our application, we assume a finite choice space.

Suppose an individual faces a choice among n different options. A boundedly rational individual is assumed to start with a random choice and adjust her choice over time in a way that appears beneficial given her current experience. From time to time the individual may also experiment. This kind of behavior usually leads to a Markov process over the choice space, which can be described as:

$$p_{t+1} = f(p_t, u), \tag{56}$$

where $p_\tau \in \Delta^n$ is the vector of choice probabilities at time τ, $u \in R^n$ is the vector of utilities associated with different choices, $f : \Delta^n \to \Delta^n$ is a continuous function and Δ^n is the $n-$ dimensional unit simplex. The steady states of equation (56) can be interpreted as long-run choice probabilities.[16] A simple form of this relationship occurs if the transition probabilities between the states of the system are constant:

$$f(p_t, u) = A(u)p_t, \tag{57}$$

where A is an $n \times n$ matrix.[17]

The steady state probabilities for equation (56) can be written as:

$$p^* = p^*(u) \tag{58}$$

Let us also impose the following symmetry condition: for any permutation $\delta : \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ one obtains

$$q_{t+1} = f(q_t, v), \tag{59}$$

where

$$q_\tau^i = p_\tau^{\delta(i)}, v_\tau^i = u_\tau^{\delta(i)}, \tag{60}$$

i.e. dynamics (56) remains invariant under the relabelling of the choices. If the symmetry condition holds system (56) should generically possess at least one symmetric steady state,[18] i.e. a steady state where the choices that have equal utilities will be made with equal probabilities.[19] Symmetry also implies there is a simple transformation of the relationship in equation (56) when a new alternative is added such that the probabilities of previously available options fall. Also, note that the probability with which each outcome is chosen depends solely on the vector of utilities.

Finally, note that, as argued by Anderson et al (2004), the Luce model can be derived directly from a Markovian learning model.

[16]Note that a steady state of system (56) exists according to the Brouwer fixed point theorem.

[17]Condition (57) is usually violated in social learning models. For examples of such models, see Basov (2006).

[18]This follows from the index theorem (see, for example, Section 17D of Mas-Colell, Whinston, Green, 1995), which implies that generically the number of the fixed points is odd.

[19]If condition (57) is satisfied then the steady state is generically unique, and therefore symmetric.

Random utility and Address Models

In random utility models it is assumed that the utility of each option is affected by a random idiosyncratic shock, which is unobservable to an econometrician. Individuals are rational and choose the option with the highest total utility, which is the sum of the observable and unobservable components. However, from the point of view of an econometrician the choice is probabilistic.

In address models probabilistic choice on market level arises from unobserved heterogeneity of the consumers in the horizontal direction. Anderson, de Palma, Thisse (1992) establish that under broad assumptions these models are equivalent to random utility models.

Hence, in the standard random utility (or address) model, as described by Anderson, de Palma and Thisse (1992), an individual chooses one of the n alternatives, with which the payoffs $u_1 + \varepsilon_1, ..., u_n + \varepsilon_n$ are associated. Vector u is publicly observable and common among the individuals. For example, if the alternatives are jobs vector y, u can refer to wages. Vector ε, on the contrary, is the private information of the individual. We assume that it is distributed over R^n with some strictly positive density, which does not depend on the base payoff vector u. An econometrician will observe the following choice probabilities:

$$p_i(y) = \Pr(u_i = \max u_j) = \Pr(\varepsilon_j \leq u_i + \varepsilon_i - u_j,\ \forall j = \overline{1,n}). \tag{61}$$

First, note that any random utility model can be re-interpreted as resulting from Markovian learning. Indeed, if $q \in \Delta^n$ is the vector of probabilities generated by the random utility model, consider a Markovian model with

$$f(p, u) = (qq^T)p, \tag{62}$$

where q^T is a row vector transposed to the column vector q. It is easy to see that vector $p = q$ is a steady state of system (56) (recall that since q is a probability vector, $q^T q = 1$). The reverse, however, is not always true. Falmange (1978) proved that a system of choice probabilities has a random utility representation if and only if its Block-Marschak polynomials are non-negative. Intuitively, non-negative Block-Marschak polynomials imply that adding an alternative never increases the probabilities of the remaining choices and the marginal effect of adding an alternative (as well as the marginal effect it has on the marginal effect, etc.) decreases as choice set shrinks (see, Anderson, de Palma, Thisse, 1992).

Second, note that the choice probabilities depend on both the utilities and the distribution of the random shocks (as there are infinitely many joint distributions of $(n+1)$ random variable with the same $n-$dimensional marginals).

Finally, the Luce model can be derived from a random-utility model if the unobserved components of utility are independently, identically distributed according to the extreme value distribution, with parameter λ (see Anderson, de Palma and Thisse, 1992). However, generating the specific result that probabilities of existing varieties decline as new varieties are introduced requires extra assumptions on the joint probability distribution of random components of utility for the new and old varieties, as the general random-utility requires only that probabilities are non-increasing.

Machina's approach

Machina (1985) proposes that individuals have direct preferences over probability distributions summarized by a function $V(p)$. Machina's approach can be interpreted as a model of bounded rationality by associating with each probability distribution a numerical function, the cost of computation.

Definition *A continuously differentiable convex function $c : \Delta \to R_+$ is called a cost of computation if*

$$\lim_{y \to x} \|\nabla c(y)\| = \infty \tag{63}$$

for any x on the boundary of Δ.

This definition implies that definitely excluding even one alternative (as represented by a point on the boundary of the unit-simplex) as a possible solution is prohibitively costly. On the other hand, selecting some distributions may entail very low cost. For example, I may look at my watch and select the alternative whose number in the list equals the number of minutes past after the last whole hour. This rule will produce some distribution of choices that depends on my behavioral habits. Though the quality of choice will be very poor, since the rule has no relation to the actual payoffs, the cost of computation in this case is minimal. A boundedly rational individual selects the vector of choice probabilities $p^*(u)$ to solve:

$$p^*(u) = \arg\max(\sum_{i=1}^{n} p_i u_i - c(p)), \tag{64}$$

i.e. Machina's utility function has a form:

$$V(p) = \sum_{i=1}^{n} p_i u_i - c(p). \tag{65}$$

For a given set of choices, any choice probabilities derived from a random utility model can be obtained from (64) for an appropriate choice of $c(\cdot)$. More precisely, the following theorem holds:

Theorem *(Hofbauer and Sandholm, 2002) Let $p^*(u)$ be the vector of choice probabilities obtained from (61), where the components of vector ε are i.i.d. over R^n with some strictly positive density, which does not depend on the payoff vector π. Then there exists a convex function $c : \Delta \to R$, continuous on Δ and continuously differentiable on its interior such that*

$$p^*(\pi) = \arg\max(\sum_{i=1}^{n} p_i u_i - c(p)) \tag{66}$$

$$\lim_{p \to q} \|\nabla c(p)\| = \infty \tag{67}$$

for any q on the boundary of Δ.

Note that, as with the Markovian model, probabilities are solely a function of the utilities. Furthermore, if we specify the cost of computation as

$$c(p) = \eta(\ln n + \sum_{i=1}^{n} p_i \ln p_i). \tag{68}$$

we can directly derive the Luce model (Fudenberg and Levine, 1998).

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